DOCUMENT RESUME

ED 259 948 SE 045 928

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TITLE	Conceptions and Representations: The Circle as an Example.
PUB DATE	Apr 85
NOTE	25p.; Paper presented at the Annual Meeting of the American Educational Research Association (69th, Chicago, IL, March 31-April 4, 19`5).
PUB TYPE	Reports - Research/Technical (143) -- Speeches/Conference Papers (150)
EDRS PRICE	MF01/PC01 Plus Postage.
DESCRIPTORS	*Cognitive Processes; Computer Oriented Programs; Computer Software; *Concept Formation; Mathematics Education; Problem Solving; Secondary Education; *Secondary School Mathematics
IDENTIFIERS	*Circles; *Mathematics Education Research

ABSTRACT

This paper, which addresses the issue of representation as an internal construct corresponding to an external abstract configuration, attempts to extend A. A. DiSessa's phenomenological primitives to mathematics (particularly to the notion of circle). Various acceptations of the word representation are examined, using the notion of a circle as an example. Primitive conceptions are presented together with two tasks aimed at probing their presence or evolution. Two computer programs (three-ring puzzle and moving-around-the arc) which bring forward the development of primitive conceptions are described as is a small-scale investigation in which the programs were used. Negative results obtained from the investigation are analyzed in view of the intrinsic difficulty of pinning down the elusive mental constructs. The importance of the research project as computers start to be used to enlarge children's universe of experimentation (micro-world) is stressed. (Author/JN)

ED259948

CONCEPTIONS AND REPRESENTATIONS:

THE CIRCLE AS AN EXAMPLE

ABSTRACT:

The article addresses the issue of representation as an internal construct corresponding to an external abstract configuration. It attempts to extend DiSessa's phenomenological primitives to mathematics (more precisely to the notion of circle). It was originally inspired from a research project conducted in France by ARTIGUE and ROBINET (1982).

The paper examines various acceptations of the word representation. The notion of circle is used as an example. Primitive conceptions are presented together with two tasks aimed at probing their presence or their evolution. Two computer programmes bringing forward the development of primitive conceptions are described. Negative results are analysed in view of the intrinsic difficulty of pinning down such elusive mental constructs. The conclusion stresses the importance of the research project while computers start to be used to enlarge children's universe of experimentation (micro-world).

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Avril 1985

SE 045 928

LESH (1979) and JANVIER (1980) studied the translations involved between various representations of the concepts of rational numbers and variables. In JANVIER (1983), we have tried to extend the notion of representation so as to encompass an internal counterpart to the visible sign. We showed that the concept of function was only unique from an axiomatic perspective. In other words, it could be broken into several non-overlapping semantic domains, such as variable, transformation, sequence, isomorphism. Differences between the domains variable and transformation were pointed out based on psychological grounds. This paper is aimed at studying further the notion of representation considering primarily its internal component. The Montreal Conference on Representation forced us to re-examine the theoritical background of a research project that was being carried out at that time.

In DISESSA (1981), we find an excellent analysis of a few phenomenological primitives (p-prims) such as springiness, squishiness, dying away... which he defines as "recognizable phenomena (basic in nature)(1) in terms of which they (the students) see the word and sometimes explain it". He highlights in his conclusions two ways by which they are involved in "expert" thinking:

> "P-prims serve as elements of analysis, we might say models, which partially explain and provide rapid qualitative analysis for similar but more formal ideas; the recognition of a p-prims can serve as a heuristic cue to other, typically more formal analyses."

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(1) Our comment.

Reasoning expertise is arrived at (according to DiSessa) through the construction of a priority system which links the p-prims betw n themselves and to the "textbook" concepts.

We believe that DiSessa p-prims are very close to the "imagistic configurations" of GOLDIN (1983). It seems likely that expert thinking in mathematics and physics is not achieved, as he puts it:

> "by superior encoding of verbal problem in formal symbols or by more efficient processing of formal notations, or by much better and more sophisticated heuristic plans".

In fact, the texts show that the three of us agree on the fact that mathematical competence can be most attributed to either the construction of a superior imagistic representational system or to a rich system of p-prims "organised by" a flexible priority system. The present paper is aimed at 1) analysing further the internal component of a representation; 2) describing tasks which probe two conceptions of circle and 3) providing the preliminary results obtained.

Representation, conception, concept

Let us first of all clarify the terminology. As noted in DENIS et DUBOIS (1976), the word representation has roughly three different acceptations in the psychology literature.

At first, representation means some material organisation of symbols such as diagram, graph, schema... which refers to other entities or "modelises" various mental processes. We recognise the usual domain of signifiers which refer to inacessible "signifieds".

The second meaning is much wider. The word according to various schools of thought has several closely related acceptations which all refer to a certain organisation of knowledge in the human mental "system" or in the long-term memory. In fact, one singles out in this meaning the more or less raw material on which cognitive activities are based. In certain cas representation can be identified with concept. In other, they are the ingredient from which they are formed.

The third meaning refers to mental images. In fact, it is a "special case" of the second one. The distinction deserves being done because of the importance of the research of this domain and also because of its clear theoritical framework.

In fact, I would classify DiSessa's p-prims as belonging to the second category as well as Goldin's imagistic configuration which could also be assigned to the third meaning. As we did in JANVIER (1983), we shall use the word schematisation or illustration to refer to the first meaning and for the first time make use of a more general vocable namely conception to refer to the second meaning.

We find in the literature expressions such as pre-conception or pre-concept which certainly relate to the same reality. We have prefered the word conception to the word concept to stress its independence with respect to an organised theory. Indeed, several authors view the concept as the mental counterpart of entities pre-existing (or existing) in a theory or within an explanatory model. In other words, concepts belong to science before getting a psychological status, they have a strong cultural connotation and are expected to be mental constructs which are shared by people in order to make communications possible. Conceptions as we understand them develop mostly outside organised theories as our examples will show.

As for the prefix "pre", we have avoided using it since we have deliberately decided not to stress a dubious anteriority in time. In fact, several protocols (CLEMENT (1979) and LOCHHEAD (1979), for instance) show that conception are essentially dynamical. They are very elusive object which are difficult to distinguish from the rules of "natural reasoning" acting on phenomelogical stimuli in conjunction with learnt principles. In fact, we regard much more a conception as an evolving entity inside a learning sequence than as a preexisting one which determines the students' responses.

What pre-exists is mostly a natural logic producing rules for induction and deduction. We shall interrupt our theoritical considerations and present an example on which future comments will bear.

Conceptions of circle

The notion of circle is among the first "geometrical shape" children recognize. As BIALYSTOK and OLSON puts it (in their own terminology):

> "round is initially part of a perceptual recognition routine which, when combined with the significance or meaning rolls and bounces yields the concept ball. When the feature(s) round is isolated from that object, to become an object of perception in its own right with its distinctive meaning, it has become a spatial concept. The new concept round is equivalent in status to the concept ball in that it can be uniquely referenced and retrieved, manipulated, transformed, and imagined as an object of thought, and used as a basis for categorization."

As clearly stated in the last line, the aim is categorisation. In fact, most studies in psychology that are reviewed do not involve "advanced" processing or handling of concepts or conceptions in relation to scriptural patterns. Roundness should, in our opinion refer more to phenomelogical or experimental features (used as basis in "reasoning").

The textbook concept of circle which is learnt at school must articulate in some manner with the initial gestaltist concept. The instruction or school concept is constructed with stencil or compas. The concept may become more analytical according to the type of instruction.

Byalystok and Olson points out rightly that one can recognize complex visual patterns and fail to identify the features which gave rise to the recognition. In other words, one may use spatial informat recognize object without being conscious of the features used.

The process of perception (and we agree with them) is guided by prior frames and is more active than passive. This is a well known fact. To explain this active role (and other reasons) they stand for a theory of categorical representation of spaces mainly achieved through the construction of "categorising" propositional form of the type: predicate (.eferent, relation) such as: "on (ball, carpet)". Let us note that in fact the usual teaching of geometry which mostly stresses with words the main features of objects play this role of guidance in the perception.

We personaly consider that consciousness is too often "equated" with verbal explanation and word (linguistic) expression. In fact, most of our prior research in the reading of graphs has precisely shown the great importance of word as cue and "beacon". However, consciousness can also relate to sequences of actions or processes with a minimal use of words. In other words, a first search for explanations might only elicit the articulation of a few basic actions or images via a minimal of tying words. This is exactly where we find "room" for the notion of conception. Indeed, for more advanced (than recognition) processing, judgments are based on present and prior experience which are used in conjunction with some "natural logic" as our examples will show.

Let us note before that we are very far from the notion of representation as a symbolic object (first meaning) and that we deal with conceptions (representations, second meaning) associated with abstract written sign (and not totally symbolical). In fact, one could try to extrapolate using analogy before experiments are achieved in the field of conceptions associated with signs more symbolic in nature.

A basic research

The experiment described below stems from an interesting research project reported in ARTIGUE and ROBINET (1982). They derived from interviews based on several items and from work done in classrooms a set of conceptions which they relate more or less in a one-to-one fashion to eleven correct definitions. However, their report does not tackle the question of characterising the notion of conception. At the most, they point out that the richness and complexity of "viewpoints" that can be associated with object otherwise uniquely defined in textbooks. They note also that "viewpoints" are often fragmentary as we shall see later. They also distinguish three criteria used in comparin the definitions and consequently (for them) the conceptions: 1) the pointwise-global opposition, 2) the static-dynamic spectrum and 3) the explicit reference in the definition to basic features such as a) the center, b) the diameter and c) the radius as length and as a segment. As for the concept of circle, the definition which is used in textbooks evolved with the appearance of set theory. Traditionally, circle was defined as a curve such that all lines drawn to the curve from a common point located inside the curve would

be of equal length. Recently, the curve became a set of points and the equal distance condition was simplified into "all at equal distance from an inside point called the center" (note that the radius as segment "vanishes"). The concept certainly stresses anyhow the center as a major distinghish feature.

Two tasks to probe conceptions of circle

A) The three-ring puzzle

The pupil is given an envelope containing all the parts but one of three card board rings each cut into seven parts (see Figure 1). The pupil is told that a part is missing but does not know from which ring. Actually, the missing part belongs to the big ring. The aim of the game is to make the puzzle. The further analysis of protocols deals primarily with the method used to complete the big ring and with the general strategy utilized in making the puzzle. There are materials on the table: some blank paper, some drawing paper, a graduated ruler, a compas, a pencil, scissors.

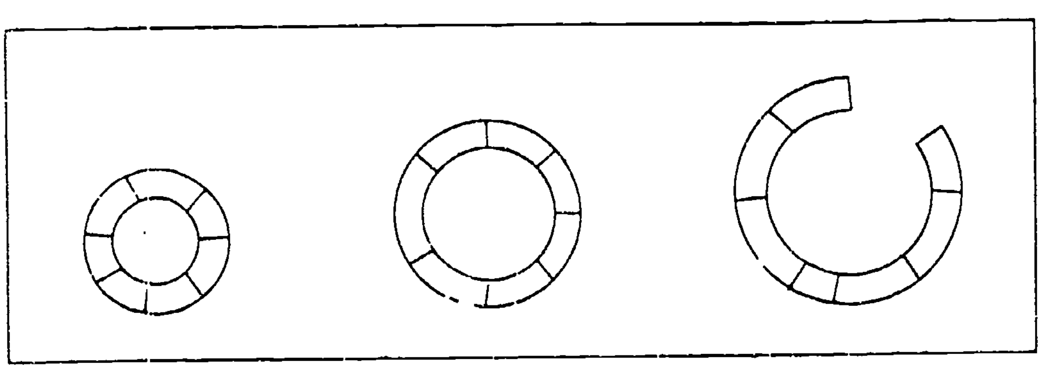

Figure 1

B) The spiral puzzle

The envelope contains the parts of a card board spiral (about ? cm wide). The pupil is asked to make the puzzle and the method used is mostly observed and analysed.

Those two tasks were used in preliminary interviews in order to verify the presence or the evolution of circle conceptions in the strategy used to make the puzzle. Questions were only of the kind: Why do you reject this part? Why do you keep this one? Is this really a circle? Why? Those interviews were conducted by graduate students of mine.

Prior experiments had also been conducted in France with recognition tasks involving figures resembling circle but which were not. Pupils had to explain why certain figures were not circle. In that case, Robinet and Artigue mainly noted the use of school concepts in order to partially characterised circle. The idea of symmetry was recurrent. We may regard this rearrangement of school concepts as a conception but we must note in that case that a conception consists in a partial characterisation expressed as a necessary (but not sufficient) condition for a closed curve to be a circle. The symmetry in all directions do not have to be expressed.

The idea of <u>unequal diameter</u> was also frequently expressed. By the way, diameter was more regarded as splitting the circle into two parts than as a double radius.

But with the puzzles, we observed that school concepts are not frequently used. Actually, expressions such as "turning even", "turning the same" were frequently heard. More precisely, the french expression used was "rentrer" which at the same time involve the idea of "being bend" and "turning". In fact, only a motion with the head or the fingers can reveal whether the arcs considered are apprehended <u>globally</u> (and are looked at as differently bend) or they are more dynamically examined and the curves are found more or less sharp.

<u>Conceptions and the tasks</u>

The preliminary interviews allowed us to define more clearly the conceptions in relation to the tasks and to plan an experimental design. We decided to make the following hypotheses.

The completion of the incomplete ring might be achieved using a compas in which case the textbook concept prevails in the reasoning. If another long piece is used and correctly place to cover the gap, we can suppose then either a global curvature conception is used or the <u>homogeneity under rotation</u> is the conception at work.

This conception stems from the remarquable fact that circles with straight line are homogeneous in all their part in the sense that they can slide on themselves without any part showing off. We shall then speak of 1) a global curvature conception, 2) a dynamical curvature conception and 3) a homogeneity under rotation conception.

The methods used to assemble the rings are expected to reveal the global curvarute conception if the strategy used consist in superimposing the arcs to check if they "fit". On the other hand, if they are connected end to end so that they appear to turn the same way than we can suppose (and only suppose) that a more dynamic curvature conception is being used or developed.

As for the spiral, we consider that the solution" would mostly reveal the use of the dynamical curvature conception.

Computer programmes developing conceptions

Since conceptions are mostly based on prior experience and mental images, we thought about trying to induce or develop circle conceptions. Both programmes finally look like game and the learning can easily be seen as imply incidental. This is the way we think conceptions should develop more implicitly than explicitly.

A) Moving round the arc into a circle

The development of this program is an interesting story by itself but we shall limit ourselves to present the end-product.

The central idea of this programme is the rotation of an arc (of circle) achieved by turning the knob of a paddle clock-wise and counter clock-wise. The relation with the homogeneity under rotation conception is patent. However, to insert this "turning" action into a meaningful task we impose conditions on the circle obtained with the paddle.

It is required to be tangent to two lines appearing on the screen (see Figure 2). Then the arc can be moved sideways, upwards and downwards before it is completed. As soon as the knob is activated the resultant circle is fixed.

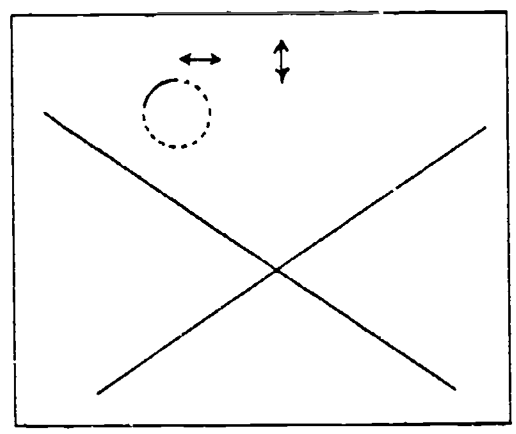

Figure 2

Various levels of difficulty are introduced and a score is given by placing the arc differently at the beginning or by replacing the two lines, with one line and a point (version 2) or two points (version 3).

B) Riding with your radius of curvature

This second computer program is not completed yet but it is aimed at developing a dynamic curvature conception of cercle. With a few contraints making it into a game, a car will be driven with a joystick and at any instant the radius of curvature will stick out perpendicular to the motion. If the car is left alone at any moment it goes on along a circular path.

Description of the experiment

We have so far carried out a small scale experiment involving the three-ring puzzle and the moving-round-the-arc programme. We spent a day in a school and worked with twelve groups of two pupils aged 13-14 for one half and 15-16 for the other half. Unfortunately, we lost (for technical difficulties) two video taped interviews and so are left with ten.

The interviewing technique is strongly inspired from BALACHEFF's (1981)"pupils' interaction technique". We consider pointless trying to check how effective the conception is prior to the treatment since it might contribute partially to develop it in an un-expected manner. We prefer to put forward the hypothesis that the conception is roughly inactive at the beginning and not use any pre-test.

The methodology is as follows. The control groups (of two pupils) work on various problems having received the assurance that they would be next on the computer and the experimental groups work two-by-two on the computers. After twenty minutes of computer work or of problem solving, <u>one</u> pupil of the group is presented with the task. He (she) is told that his performance does not count. In fact, when he (she) will have completed the puzzle, he (she) will explain to his (her) partner what to do. His (her) partner's time will be the one that will count. When the puzzle is solved, the second one come in and listen carefully to his friend. A chronometer is then used to time the second pupils's performance (but for us, it has no value!).

<u>Preleminary results</u>

The variety of responses obtained turned out to involve several factors which the pre-experiment had not allowed us to pin down.

a) The puzzle can be more or less envisaged by the pupil as a school activity which he (or she) must succeed and not as a game. At any rate, the desire to solve the problem quickly brings about a nervous tension which is incompatible with an open search for alternative methods.

b) There exist an ambiguity in what the final configuration of the ring should look like. In fact, we naively assumed that rings should be circular. They happened to be formed for pupils like ellipses.

c) The explanations from one pupil to his (her) partner turned out to be no more than a restatement of the nature of the game. Pupils could not explain "in words" what they have done. A few expressions however were made which we used in order to characterize conception more fully.
Remark: Next time, the interviewer will explain the game and the pupil will start his (her) method from this point.

d) The systematicity of the approach induces various degrees of complexity to the task. If only one ring is formed at the same time we see that we have six "different" games. If the rings are constructed in turn then the multiplicity of paths to the final pattern is obvious. More particularly, if the big one is tackled first and the missing part is hopelessly searched in the pile.

e) Consequently, one response is in fact a succession of quick decisions for using a strategy or to switch to another, each decision being taken all the time from a different set of "initial conditions". This simple fact makes the ultimate or final strategy used basically incomparable and even casts some doubt on comparisons we would be tempted to make between the approach used to complete the big ring.

In view of those facts, results are more than correctly qualified as preliminary. We shall present in turn the completion of the big ring and the first strategies. As explanations of one pupil to the other appeared to have not too much effect, their responses were not analysed.

The completion of the big ring

Out of the five pupils experimental group, three started using a compas, two changed their mind to use a part to fill the gap. One simply draws the missing part. The fifth one uses a part of the big circle to complete (free hand) the medium side one.

Of the control group, four pupils used a ccmpas and one draws a missing part free band on a piece of paper and cut it with a pair of scissors.

Discussion and interpretation

We would be enclined not to attach to much importance to this difference in favour of the experimental group mainly because of the great diversity observed in the pupils getting their rings done as mentioned before. In fact, in a few cases we had to intervene.

Consequently, we believe that the homogeneity-under-rotation conception was not enough present to dislodge the school concept based on compas. In a further investigation we think that pupils should play the game for more than twenty minutes and on a few occasions.

One difference appeared more clearly. One out of four grade 8 pupils used a compas as the final method to complete the circle while for grade 10 the ratio is 5:6.

The first strategy

We will give a general description of the strategy observed and discuss the difficulty we have to assign conceptions to them. Eight of the ten pupils resort at first to the puzzle-making strategy which consist in connecting the parts end-to-end. Rejection or acceptation of each part was then based (if we rely upon reactions to our interventions, pre-experimentations and comment from one pupil to the next) on either a dynamic strategy witnessed by statement such as: "turn too much", "turn the same" or on a more static one even though "end-to-end" suggests motion. In the last case, the curve seems to be examined more globally. The pupils refer then to curves being "more or less pronounced" or to "not round enough"(1), "too much round"(2). The word bend is not as such often associated with curve. We imagine that this word in english would reveal a

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(1) (2) Very approximate translation of "pas assez rond" and "trop rond" which are not even correct in French.

more static approach to the curvature.

Side by side examination clearly indicates a more global approach. It was used by a few pupils after failure with a end-to-end strategy. However, it does not garantee success since parts are not of the same length and comparisons involved difficulties when made with small parts.

Superimposition was used by only two pupils. It is not easy to pin down the conception behind this approach. Indeed, the curvature can be appreciated dynamically before the comparison is achieved or it can be perceived more globally. Moreover, it is even impossible that a "homogeneity-under-rotation" conception be used.

Only one thing remains certain: ther. does not seem to be connection between end-to-end and the use of compas to complete the big ring. Indeed, we observe superimposition followed by the use of compas twice and end-to-end leading three times to completion using a part.

Discussion

As we are planning to carry out the experiment once again shortly, it seems appropriate to indicate the major expected changes. As already said, the pupils will have the opportunity to play the game several times and over a longer period of time. We will keep our original pupils' interaction methodology and, as mentioned, the interviewer will explain the rule to the second pupil. We intend not to analyse systematically the

second pupil's response. However, we have already establish a list of blocking points to which we hope to arrive at a systematic reaction from the interviewer. Our intention is to focus our attention in our analysis on a few similar patterns of response in order to formulate hypotheses on the coherence of conception systems and their evolution. We hope that a large number of responses will bring us those few similar ones. More attention will be paid to the actions (gestures) made by the pupil. We will wonder if his attemp to "deverbalize" our study of conception is not presomptuous since, as we said, conceptions are very elusive entities which evolved and, consequently, cannot easily be regarded as present at one time. As for the computer programmes themselves, we do not expect any changes.

Conclusions

ARTIGUE and ROBINET (1982) paved the way to the analysis of various conceptions related to the notion of circle. We tried to examine the status of conceptions in relation to DiSessas's and Goldin's work. Not only do we believe that conceptions play a central role in translation processes and mathematical thinking but as DiSessa, we believe that they can be considered as the basis to the development of more formal ideas sometimes appearing late in the curriculum. For example, to become a textbook concept, the notion of curvature requires being wrapped in derivatives and disguised in a vectorial language. Clearly, in a problem solving situation the characterisations of the conceptualised notion are

called into play. However, we think that the conceptions in terms of mental images or anticipatory actions also intervene efficiently.

The main questions regarding the development of conception concern the timing. In fact, our experience as a teacher has shown several times that the introduction of a simplifying symbolism frequently resulted into a difficult go-back to the original spelled-out word description. Think about the logarithm, the exponentiation, the trigonometric function. As soon as formulae are provided, the meaning appears to vanish. We can easily imagine that such is the case with conceptions fading out because of "powerful" concepts. That is why we advocate conceptions should be develop at the pre-concept stage, in other words prior to the learning of the formalised concept.

This position has strong educational repercussions. On the one hand, we agree with DiSessa that they can be used as "heuristic cue to other, typically more formal analyses". But on the other hand, they could be used as the basis for a curriculum-for-all. The debate on the nature of a core curriculum is now intense in most developed countries. The right of the population to get the best must be balanced with the need for a basic instruction for all. The approach to this question is traditional in the sense that concepts, usual algorithms and processes are the objects rearranged in diverse pattern by curriculum designers. We propose that more attention be paid to conceptions for they would ensure the basic knowledge for all that would be later systematized and formalised.

As micro-computers invade schools and homes, we enter into a new educational era. To use a trendy word, micro-worlds can now be open to pupils which enable them to explore rational systems through hypothesis testing based on inductions or deductions. Using simulation and computer's pattern recognition capability, mocro-worlds provide the pupils with an interactive support to develop "explicative" and predictive models characterized by mental images and actions (as well reasonning!). The computer programmes we have decribed partially fill this goal.

We hope we have illustrated how conceptions can enter into the design of software even though we have not demonstrated that they were effectively developed.

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